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The cracking of composites consisting of discontinuous ductile fibres in a brittle matrix – effect of fibre orientation

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The effect of the orientation of metal wires on the opening of a crack in a brittle-matrix composite has been studied. The force arising from the plastic bending of a wire which is weakly bonded to the matrix and which crosses the matrix crack at an angle θ to the crack face normal has been measured in model resin-wire composites and good agreement is found with a simple theory based on the calculation of the force needed to produce a plastic hinge in a cantilever beam. The force passes through a maximum at a small crack opening, of the order of one wire diameter, and decreases with further crack opening. The The largest force is obtained for a value of θ of approximately 45°. For wires whose length approaches the critical length, the force and the total work of fracture arising from the bending of the wire are small compared to the values arising from the interfacial shear stress resisting pull-out; the contributions due to bending and interfacial shear stress are of comparable magnitudes for wires which are approximately one-fifth of the critical length.

1. Introduction

When a matrix crack is bridged by discontinuous, weakly bonded fibres, further extension of the crack is inhibited as energy has to be supplied to operate the following mechanisms: (i) fibre debonding [1]; (ii) fibre pull-out against interfacial tractions [2]; (iii) Deformation of the fibres which lie at an oblique angle to the crack face [3].

The last mechanism has been described for the case of ductile fibres by the propagation of a plastic shear along the fibre in the region of the bridge as the crack opens [3, 4]. However, there is little experimental evidence to support this theory.

In composites, such as steel-wire reinforced cement, fibre-matrix interfacial stresses are likely to be small [5] so that inhibition may depend upon the effectiveness of mechanism (iii).

This paper describes a series of experiments on model composites which simulate matrix cracks with fibre bridges inclined at an angle θ to the normal to the crack face. The load transferred across the crack was observed for various metallic fibres in resin matrices as a function of the fibre inclination and the crack opening displacement. It was found that the previously suggested shearing mechanism [3, 4] did not describe experimental observations. A new theory is proposed, based on fibre bending and matrix local yielding. The force transmitted due to fibre misalignment is compared to that due to interfacial shear stress in the case of aligned fibre, and various predictions of the theory are discussed.

2. Experimental

2.1. Introduction

An experimental model composite was fabricated from two wires symmetrically arranged at an angle 2θ in a polymer matrix (Fig. 2). The position of a deep cut sawn into the matrix defined the level at which the model cracked and the length of fibre subsequently pulled out of the resin. Prior to casting the wires were coated with release agent to minimize the effects of interfacial friction as the wires were withdrawn from the matrix.

2.2. Fibre properties

Tests were performed on models with three different metals, copper, nickel and steel. Copper and nickel wires were received in an annealed condition. These were fully coldworked prior to inclusion in the resin. This allowed a simulation of the ideal elastic-plastic fibre used in the theory of the pull-out process. The steel was of the high tensile type and was incorporated in the as-received condition. The fibre length was 20 mm and diameters ranged from 0.45 to 1.11 mm, larger diameters being preferred as they produced larger pull-out loads. A summary of the fibre properties is given in Table I.

TABLE I

Fibre	Diameter (mm)	Modulus (kN mm^{-2})	Tensile strength (N mm^{-2})
Nickel	0.45	210	530 ± 5
	1.11	210	559 ± 7
Steel	0.60	210	2030 ± 21
Copper	0.75	122	248 ± 4

2.3. Matrix properties
A polyester resin matrix was chosen. Crystic 191-E, as supplied by Scot Bader Co Ltd, was cast at room temperature with the formulation, 100 parts by weight resin, 2 parts by weight catalyst and 2 parts by weight accelerator. Specimens were post-cured at 50°C for 24 h prior to grinding and notching. The properties of the matrix so-cast are summarized below in Table II.

TABLE II

Tensile strength	48 N mm^{-2}
Modulus	3.8 kN mm^{-2}
D.P.N. hardness	26 kg mm^{-2}
Elongation	1.5%

2.4. Mechanical testing
Specimens were fractured in tension on an Instron testing machine. The load required to withdraw the fibres from the matrix was recorded. The birefringent properties of the matrix allowed observations of stress concentrations in the specimen during fibre pull-out.

3. Results
3.1. Straight pull-out tests
Specimens of fibre orientation 0° were tested to allow an estimate of the frictional effects at the fibre-matrix interface. Little could be done to eliminate this contribution probably because it is due to resin shrinkage during composite fabrication [7]. The interfacial shear stress could be reduced reproducibly to a value of about 2.5 N mm^{-2}. The method of estimating the frictional contribution to the pull-out force for inclined fibres is described below (Section 3.3).

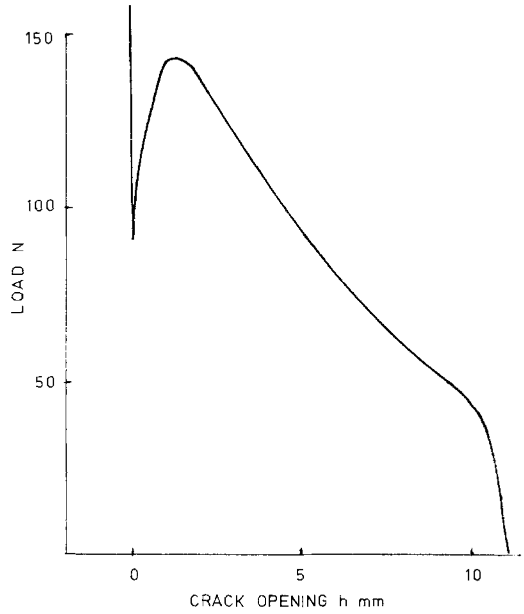


Figure 1 The variation of load with crack opening for two nickel wires, of diameter 1.11 mm and orientation 20°, in a polyester resin matrix.

3.2. Inclined-fibre pull-out
A typical load-separation curve is shown in Fig. 1. After matrix cracking, the load rises quickly to a maximum at a small crack opening and then falls slowly as the crack separation (h) increased. The position and magnitude of this maximum is of some importance as it represents the greatest bridging effect of a fibre on an opening crack.

The small value of the crack opening at which the load maximum occurred was difficult to measure accurately. It was of the order of, but rather less than, one fibre diameter.

Observations of stress birefringence showed that, even for fibre inclinations of a few degrees, substantial stress concentrations occurred in the matrix near the exit point of the fibre. Fig. 2 shows the fibre deformation and matrix stress concentrations for steel wires inclined at an angle of 24°. The wires have plastically deformed the matrix, reducing the angle through which they need to be bent. The plastic deformation of the matrix near the exit point is shown by the residual stress birefringence observed after

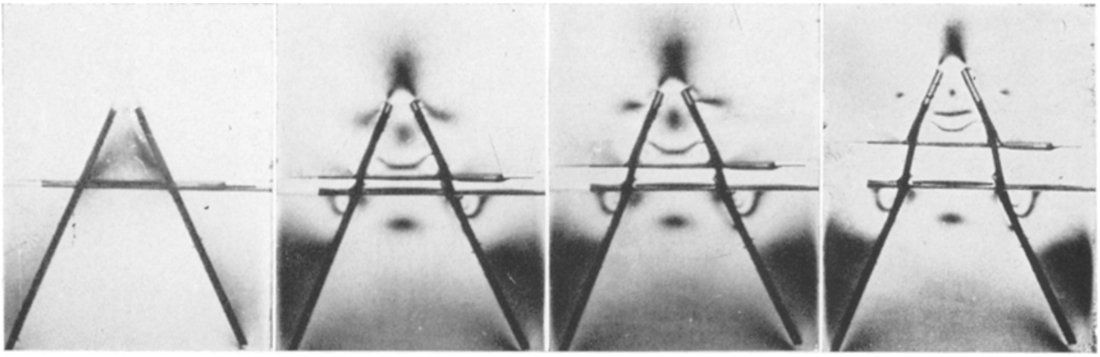


Figure 2 Matrix stress concentrations during pull-out of steel wires of diameter 0.6 mm and orientation 24°. (A sodium light source was used on a bright-field polariscope.)

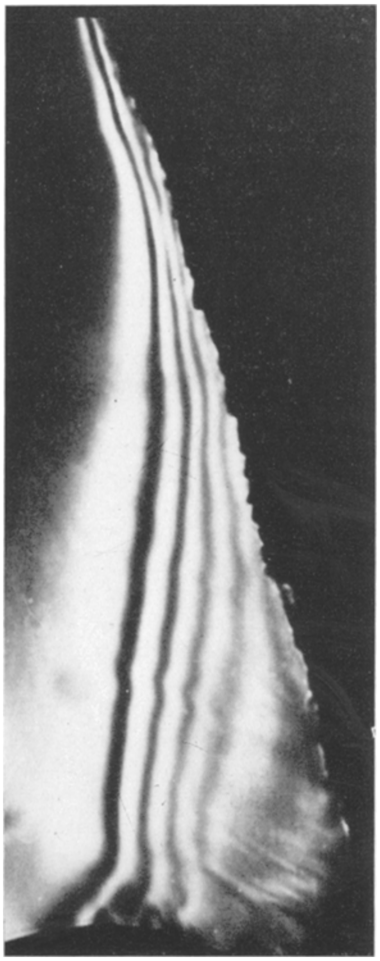


Figure 3 Residual matrix stresses near the fibre exit point are shown by the birefringent properties of the resin, using a white light source.

complete fibre withdrawal (Fig. 3). Although the nickel and copper wires produced less matrix deformation than the stronger steel wire, some was always detectable.

3.3. Measurement of the plastic work contribution

In order to estimate the frictional contribution to the pull-out force, series of tests on inclined-fibre specimens were carried out using different embedded lengths. Fig. 4 shows a plot of the load maximum during pull out as a function of embedded length. The difference between the value for an inclined fibre and an aligned fibre ($\theta = 0$) for a given embedded length gives the contribution arising from fibre inclination. It is possible, within the experimental scatter, to represent this difference as a constant, independent of embedded length. The results are, therefore, consistent with the assumption that the total maximum pull-out force consists of a component due to friction, or interfacial shear stresses, identical to that acting on a similar aligned wire of the same embedded length, together with a component arising from the deformation processes occurring near the exit point of the wire. Following this assumption, an average value of the contribution of deformation to the maximum pull-out force is obtained from the difference between the lines of load maximum versus embedded length for inclined and aligned wires. Fig. 5a and b shows the results for 0.75 mm diameter copper wires and 0.45 mm nickel wires at various inclinations. For all wires, the deformation contribution to the maximum pull-out force increased sharply with increasing inclination up to about 45°, and then decreased

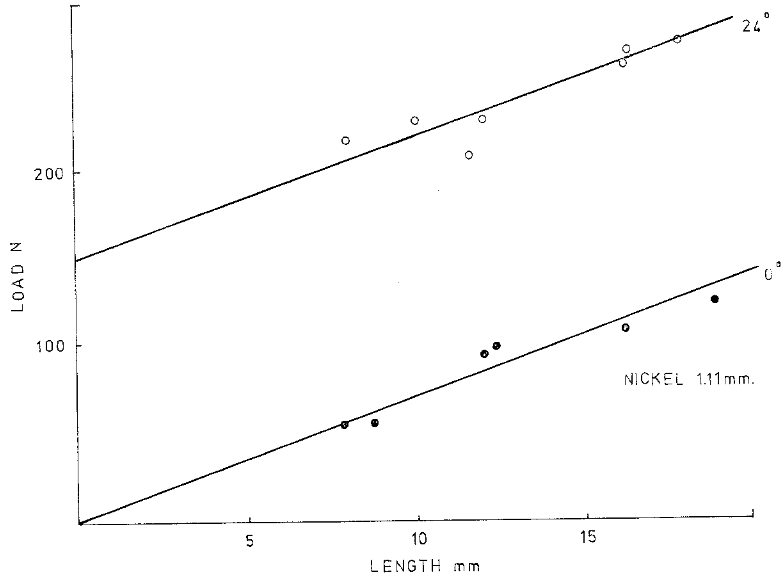


Figure 4 The variation of the maximum load during pull-out is compared for two parallel and two inclined fibres of nickel (diameter 1.11 mm) as a function of the total fibre length withdrawn.

again (Fig. 6a, b and c). Values for large angles of inclination were difficult to obtain because the matrix tended to fracture near the exit point of the wire.

4. Theoretical model
4.1. Introduction

A simple model based on elementary beam theory has been developed to account for the contribution of deformation to the pull-out force for a weakly bonded, inclined fibre. The essential geometry, based on the experimental observations, is shown in Fig. 7a. There are two important points. The first is that, because the fibre-matrix interface has a negligible tensile strength, the fibre is able to come away from the matrix over a distance $d\tan\theta$, where d is the fibre diameter and θ the fibre inclination. The second is that it is observed experimentally that the matrix always yields in compression near the exit point of the fibre allowing further deflection of the fibre within the matrix, to such an extent that the assumption of a rigid matrix would clearly be unsatisfactory. In Fig. 7, a yielded zone of length q is shown. These factors increase the length of fibre undergoing deformation, making it reasonable to apply simple beam theory to the problem. The mechanical model derived from this geometry is shown in Fig. 7b. A length of fibre from the matrix to the inflection point, assumed to be midway between the crack faces, is shown as a cantilever beam. The fibre-matrix interface is assumed to be frictionless, and the deflection of the matrix in the yield zone is assumed to be sufficiently small that all forces act in a direction normal to the fibre axis. The free length of the beam a in terms of the crack opening h is given by

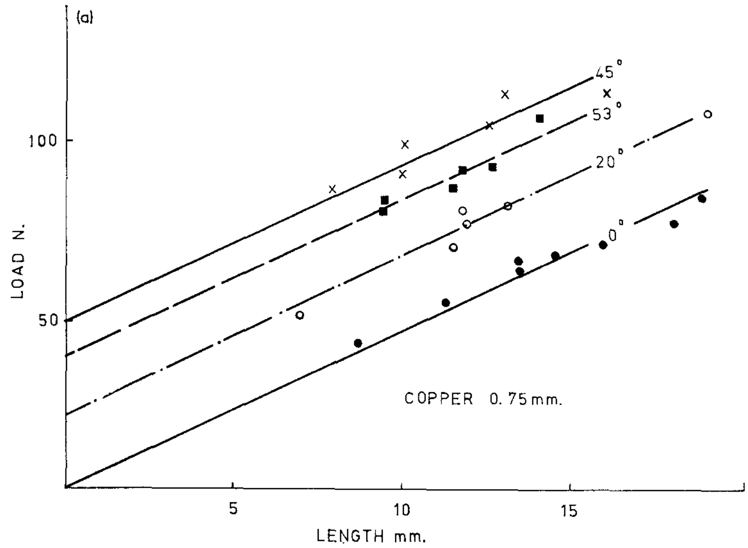
$$a = \frac{h}{2}\cos\theta + \frac{d}{2}\tan\theta\,. \quad (1)$$

The length q, supported by plastically yielded matrix, is represented by a constant pressure giving a force w per unit length of beam. Beyond the zone of matrix yield the beam is assumed to be rigidly encased. The force F is then the force needed to deflect the beam by the amount $(h/2)\sin\theta$ at its end (the reflection point of the fibre). The pull-out force is given by the component of F along the tensile axis.

4.2. Elastic deformation of the fibre

Assuming that the beam behaves elastically, and applying elementary beam theory, we have for the bending moment M_b (Fig. 7)

$$M_b = EI\frac{d^2y}{dx^2} = Fx - \frac{w}{2}\langle x - a\rangle^2 \quad (2)$$



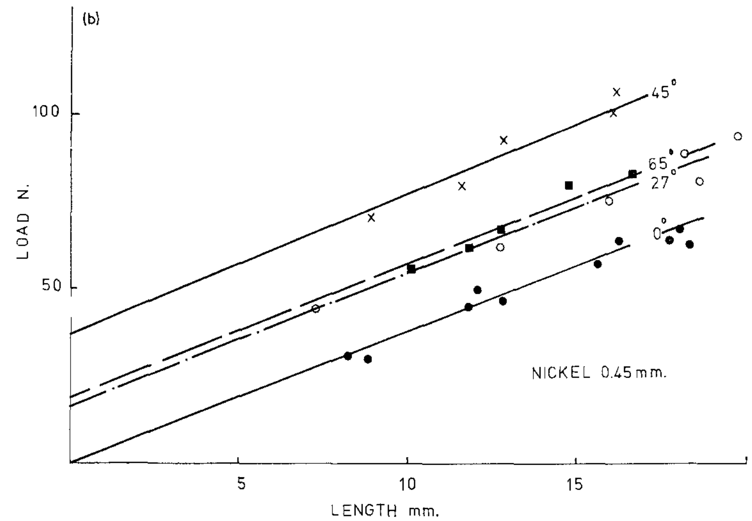


Figure 5 Variations of the maximum load required to withdraw two fibres in polyester resin as a function of orientation and total fibre length withdrawn: (a) copper of diameter 0.75 mm, (b) nickel of diameter 0.45 mm.

where $\langle x - a \rangle$ vanishes for $x < a$. The Young's modulus of the beam is E and I is the second moment of the beam cross-sectional area, $= \pi d^4/64$ for a circular cross-section.

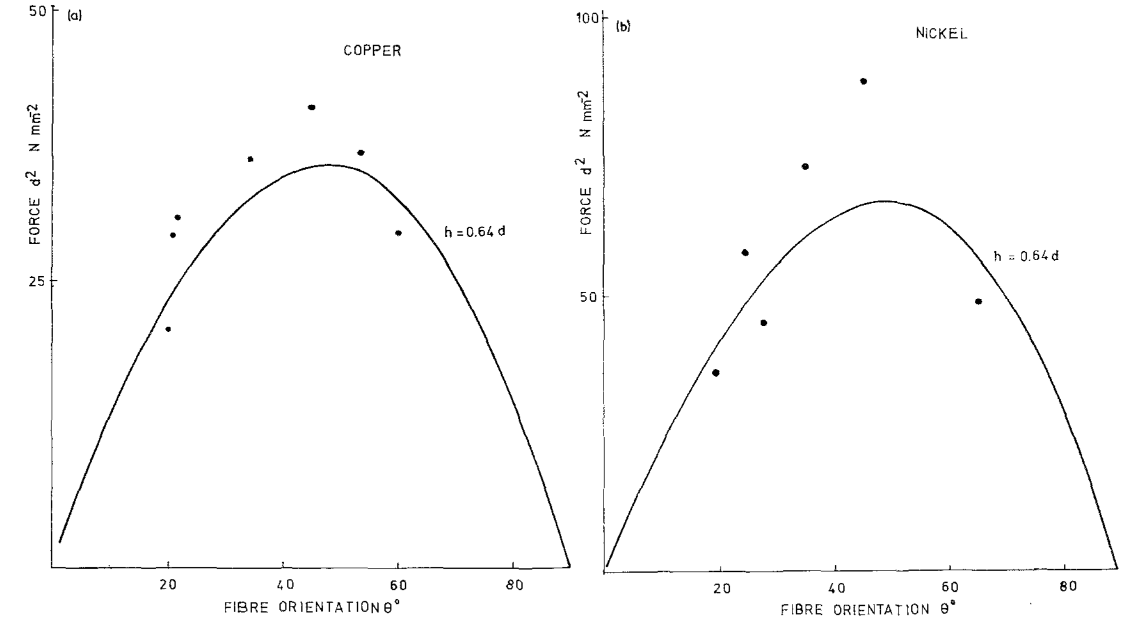
Integrating Equation 2 twice and applying the boundary conditions $y = \mathrm{d}y/\mathrm{d}x = 0$ at $x = a + q$ we have for the deflection δ at $x = 0$

$$EI\delta = \frac{F}{3}(a + q)^3 - \frac{wq^4}{8} - \frac{wq^3a}{6}\,. \qquad (3)$$

To proceed to a complete solution, a relationship between q and F is required. We shall set

$$F = wq\,. \qquad (4)$$

We justify Equation 4 on the grounds that it gives the smallest value of F for a given deflection δ subject to the condition that the shearing force at the built-in end of the beam, i.e. at $x = a + q$, shall not be in such a direction as to produce a positive slope of the neutral axis at $x = a + q$



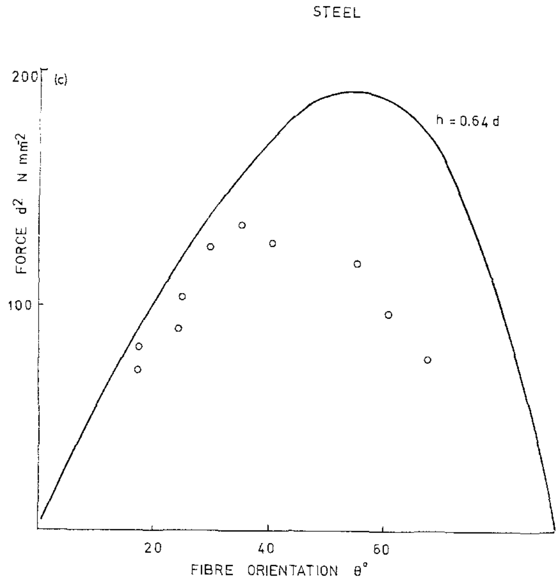


Figure 6 Experimental and theoretical values for the maximum load transmitted across a crack in polyester resin by a single fibre in a composite of zero bond stress are compared as the inclination of the fibre varies. (a) copper, (b) nickel, (c) steel. The curves are drawn from the predicted values of the load transmitted at a crack opening of 0.64 fibre diameters, using the theory described in Section 4.3.

(referring to Fig. 7b). Such a slope would appear to be inconsistent with the assumption that the beam is supported by a yielded matrix at values of x just less than $a + q$.

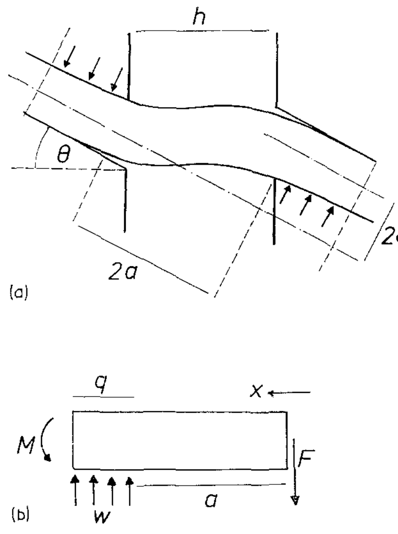


Figure 7 (a) Idealization of a fibre bridging a crack at an angle θ when the crack opening is h. (b) Beam theory forces and bending moment used to approximate fibre bridging loads.

Using Equation 4, Equation 3 becomes

$$\frac{EI\delta}{wa^4} = \frac{\beta}{3} + \beta^2 + \frac{5}{6}\beta^3 + \frac{5}{24}\beta^4 \qquad (5)$$

where $\beta = q/a$. Equation 5 may be solved for particular fibre-matrix combinations (E and I

are fibre properties but w is a matrix property) giving β and hence F for various values of h and θ. The choice of the value of w is somewhat problematical. We have taken w to be related to the indentation pressure H of the matrix as given by a Vickers hardness test. Thus

$$w = Hd\,. \tag{6}$$

The values of β given by Equation 5 show that the effective beam length $a + q$ is several times the fibre diameter even for small crack openings. Fig. 8 shows that the calculated effective beam lengths for nickel wire in polyester resin, are sufficiently large for the application of beam theory, in which shear deflections are neglected, to be valid at crack openings of the order of the fibre diameter or even less.

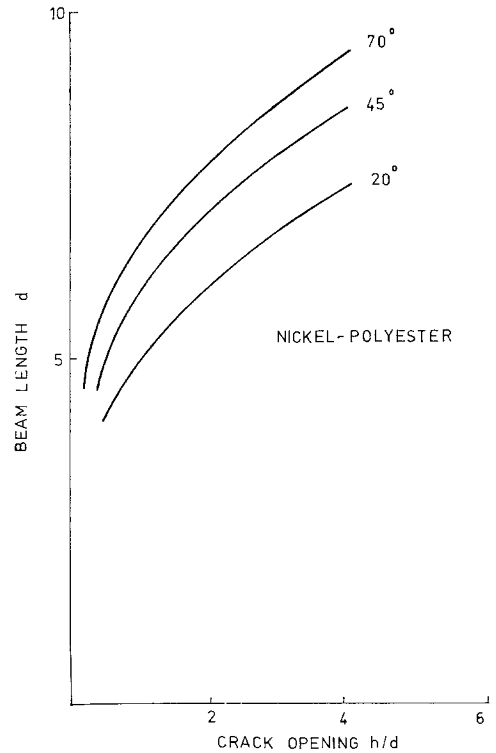


Figure 8 Theoretical estimates of the total beam length as a function of crack opening for various nickel fibre orientations in polyester resin.

Values of $F \sin \theta$, the resolved pull-out force, as a function of crack opening h have been derived from Equation 5. They show that for all cases corresponding to our experimental results, the outer fibre stress at the position of maximum bending moment reaches the yield stress of the fibre when h reaches a value which is still much smaller than d.

4.3. Plastic deformation of the fibre

From Fig. 7b, the bending moment at the position x is given by

$$M_b = Fx - \frac{w}{2}\langle x - a\rangle^2\,. \tag{7}$$

The bending moment, therefore, reaches a maximum at the position

$$x = a + F/w\,. \tag{8}$$

Note that this result is independent of the validity of Equation 4. The maximum bending moment M_{max} is given by

$$M_{max} = Fa + \frac{F^2}{2w}\,\cdot \tag{9}$$

Consider the case that the fibre becomes fully plastic at the position of maximum bending moment, i.e. a plastic hinge develops there. The fully plastic bending moment for an ideally plastic beam of circular cross-section is given by [6]

$$M_p = \frac{\sigma_y d^3}{6} \tag{10}$$

where σ_y is the tensile yield stress. From Equation 9

$$F^2 + 2waF - \sigma_y d^3 w/3 = 0\,. \tag{11}$$

The force transmitted across the crack is then $F \sin \theta$, where F is obtained by solution of Equation 11, taking the positive root. Fig. 9 shows an example of the result of this calculation, for the case of nickel at 20°. The agreement with the experimental result, corrected for the friction contribution, is good for values of crack opening greater than about one fibre diameter.

4.4. Effect of fibre inclination

The effect of the fibre inclination θ on the force transmitted across the crack by a fully plastic fibre depends on the resolving factor $\sin \theta$ and on the dependence of a on θ, given by Equation 1. Of particular interest is the dependence on θ of the maximum force transmitted during pull-out. The point of intersection between the elastic and fully plastic force-crack opening lines (Fig. 9) overestimates the maximum force, which must in reality occur during a smooth transition from

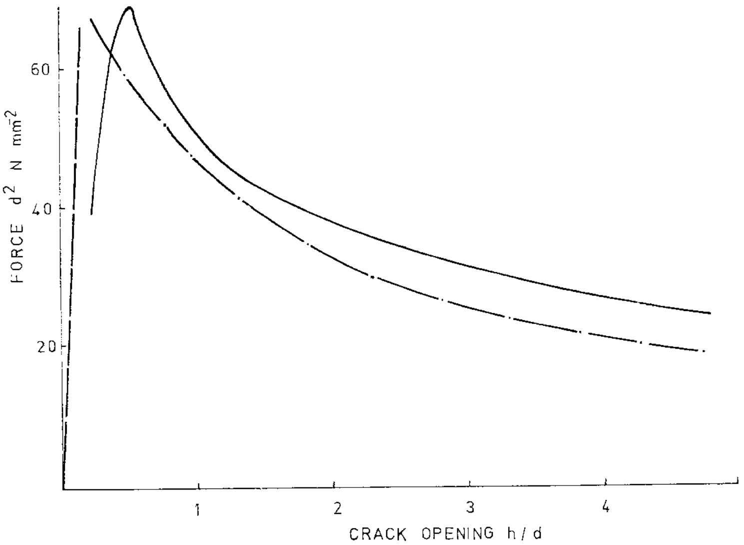


Figure 9 Experimental and theoretical values of the pull-out force due to deformation of a single nickel fibre bridge of diameter 1.11 mm and orientation 30°, as a function of crack opening.

elastic to fully plastic behaviour. A more satisfactory estimate is obtained by taking the fully plastic value at a crack opening corresponding to the position of the experimentally observed maximum. Although this position is not known with precision, the calculations can be performed for a range of values corresponding to the experimental uncertainty, and the results compared with experimental force maxima. Fig. 6a, b and c shows that satisfactory agreement with experiment is obtained, except for the case of the steel wire at larger values of θ. The disagreement here is probably due to the large extent of matrix plasticity which occurs such that the matrix-fibre interface in the yield zone does not remain approximately parallel to the original fibre axis. Specimens with $\theta > 60°$ were in all cases difficult to test as the matrix tended to fracture rather than deform plastically.

5. Discussion

It is interesting to compare the magnitude of the maximum pull-out force at the optimum fibre inclination of 45°, arising from plastic deformation of the fibre, with that obtainable from more strongly bonded fibres aligned parallel to the tensile axis. The maximum force supported by a strongly bound or very long fibre is given by its U.T.S. Thus for copper, the U.T.S. of 250 N mm^{-2} may be compared with the maximum observed pull-out stress arising from the bending of an unbonded fibre at $\theta = 45°$ of 53 N mm^{-2}. For nickel the corresponding values are 530 and 116 N mm^{-2}. For these fibre-matrix combinations, therefore, at best about 20% of the maximum force for strongly bound fibres can be obtained. This prediction may be compared with that derived from the assumption that the fibre deflects by shearing plastically at the exit point [3, 4]. Assuming a shear yield stress of $\sigma_Y/2$ and no work-hardening, the plastic shearing model gives a force resolved normal to the crack face

$$F \sin \theta = \frac{\pi d^2}{8} \sigma_Y \sin \theta . \tag{12}$$

The force given by Equation 12 is substantially larger than that given by the plastic hinge model, at a crack opening on the order of $d/2$, showing that the necessary deflection of the fibre can be expected to occur by plastic bending rather than shear. Further, serious defects of the shear model are its inability to predict the decrease in pull-out force with increasing crack opening and the decrease in maximum force for fibre inclinations in excess of 45°. Both of these observations are accounted for by the cantilever bending model developed above.

The work of plastic deformation performed in withdrawing an inclined fibre may be calculated from the area under the load-crack opening curve (Fig. 9). The calculated and experimentally determined work done in opening the crack by five fibre diameters is shown in Fig. 10 for copper and nickel wires in a polyester resin. The maximum occurs at a larger value of θ than does the largest pull-out force, because at larger values of θ the force decreases more slowly with the crack opening after passing through its maximum.

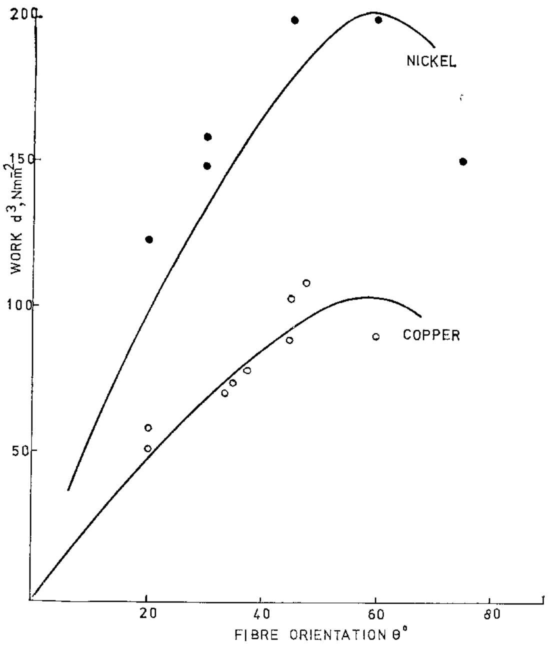


Figure 10 The plastic work component of the work of pulling out a length of fibre equal to five fibre diameters as a function of fibre orientation. Solid lines are calculated, points are experimental values.

The total work of fracture due to pulling out aligned fibres against an interfacial shear stress may be compared with that arising from plastic deformation, in the case of inclined fibres. In both cases the work done per unit area of composite fracture surface is proportional to the volume fraction of fibre V_f and the fibre diameter d, and increases with the aspect ratio of the fibres, which are assumed to be all of the same length, and to be intersected at random points along their length by the matrix crack. Fig. 11 shows calculated curves of the work of fracture per unit area, divided by $V_f d$, against fibre aspect ratio. Curve A gives the plastic work for nickel wires in polyester resin inclined at $\pm$ 45° to the fracture surface and curve B is the pull-out work for aligned fibres given by the equation [8]

$$w = \frac{V_f \sigma_f}{12} \frac{l^2}{l_c} \,. \tag{13}$$

In Equation 13, l is the fibre length and l_c the critical length, above which some of the intersected fibres are fractured at the stress σ_f, rather than pulled out. Curve C is the calculated total work of fracture for fibres inclined at $\pm$ 45° to the fracture surface, based on the assumption that the work done against the interfacial shear stress is the same as for an aligned fibre, i.e. curve C is the sum of curve B, reduced by a factor of cos 45° to allow for the smaller number of fibres intersecting the fracture surface in the inclined fibre case, and curve A. It can be seen that the contribution of the work of plastic deformation, roughly speaking, compensates for the reduced number of fibres intersecting the fracture surface, and that it is relatively more important for fibres much shorter than the critical length.

An interesting but as yet untested prediction of the theory of Section 4.3 is that an increased matrix hardness increases the pull-out force. The predicted effect for the nickel wire at an inclination of 45° is shown in Fig. 12.

It may be possible to apply some of the findings of the present work to composites other than metal fibre in resin matrixes. We have performed a short series of experiments on metal fibre-cement matrix composites. Although the results showed too much scatter to allow a quantitative comparison with the theory of Section 4.3, the force-crack opening curves were similar to those found for resin matrixes. Displacement of the matrix around the fibre exit point again occurred but in this case by micro-fracturing rather than plastic flow.

6. Summary and conclusions

The additional forces needed to open a matrix crack due to the inclination of weakly bonded, ductile fibres which bridge the crack have been measured in model composites. A simple theory based on calculating the force needed to produce a plastic hinge in the fibre as it is withdrawn from the matrix accounts for the magnitude of the additional pull-out force and its dependence on crack opening and fibre inclination.

The largest additional force occurs for a fibre inclination of approximately 45° at a crack opening of rather less than one fibre diameter. Even in this case the magnitude of the force is relatively small, being of the order of 20 % of the

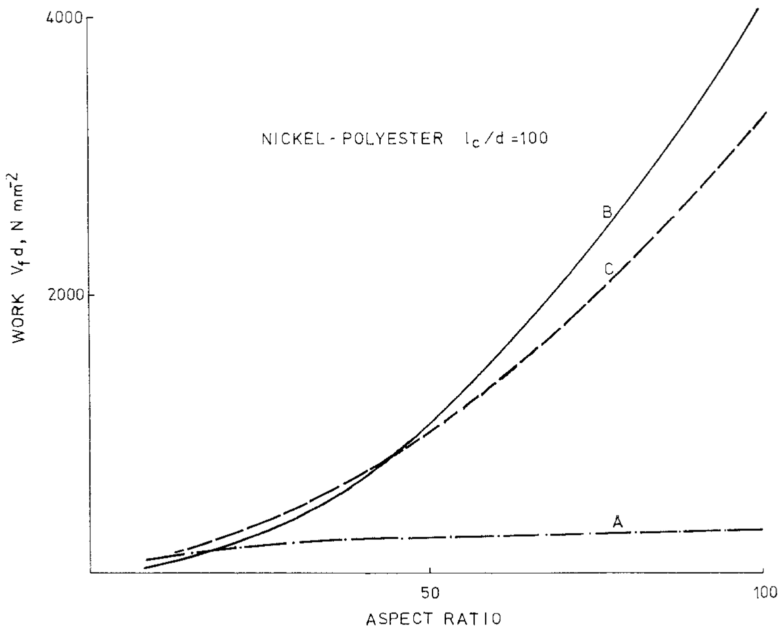


Figure 11 Calculated work of fracture of a polyester resin containing weakly bonded nickel wires. Curve A: plastic work contribution for wires inclined at $\pm$ 45°. Curve B: work done against interfacial shear stress, assumed constant, for aligned wires. Curve C: total work of fracture for wires inclined at $\pm$ 45°.

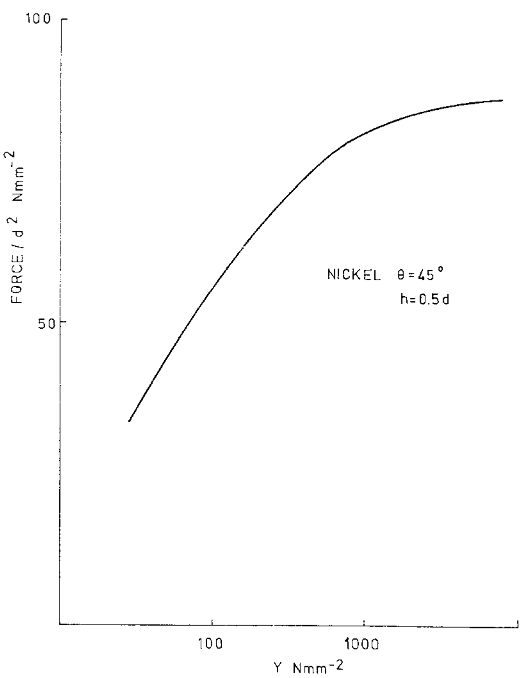


Figure 12 The predicted effect of matrix hardness Y on the maximum pull-out force for a nickel fibre inclined at 45° to the crack.

tensile strength for the cases of copper and nickel fibres in resin. Correspondingly, the work of fracture due to deforming a fibre as it is withdrawn is only a small fraction of the work done against interfacial shear stresses in withdrawing a fibre of critical length (i.e. of the shortest length which can be broken by the interfacial stress). The deformation of an inclined fibre makes an important contribution to the crack-bridging force and the total work of fracture when the fibre is much shorter than the critical length.

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